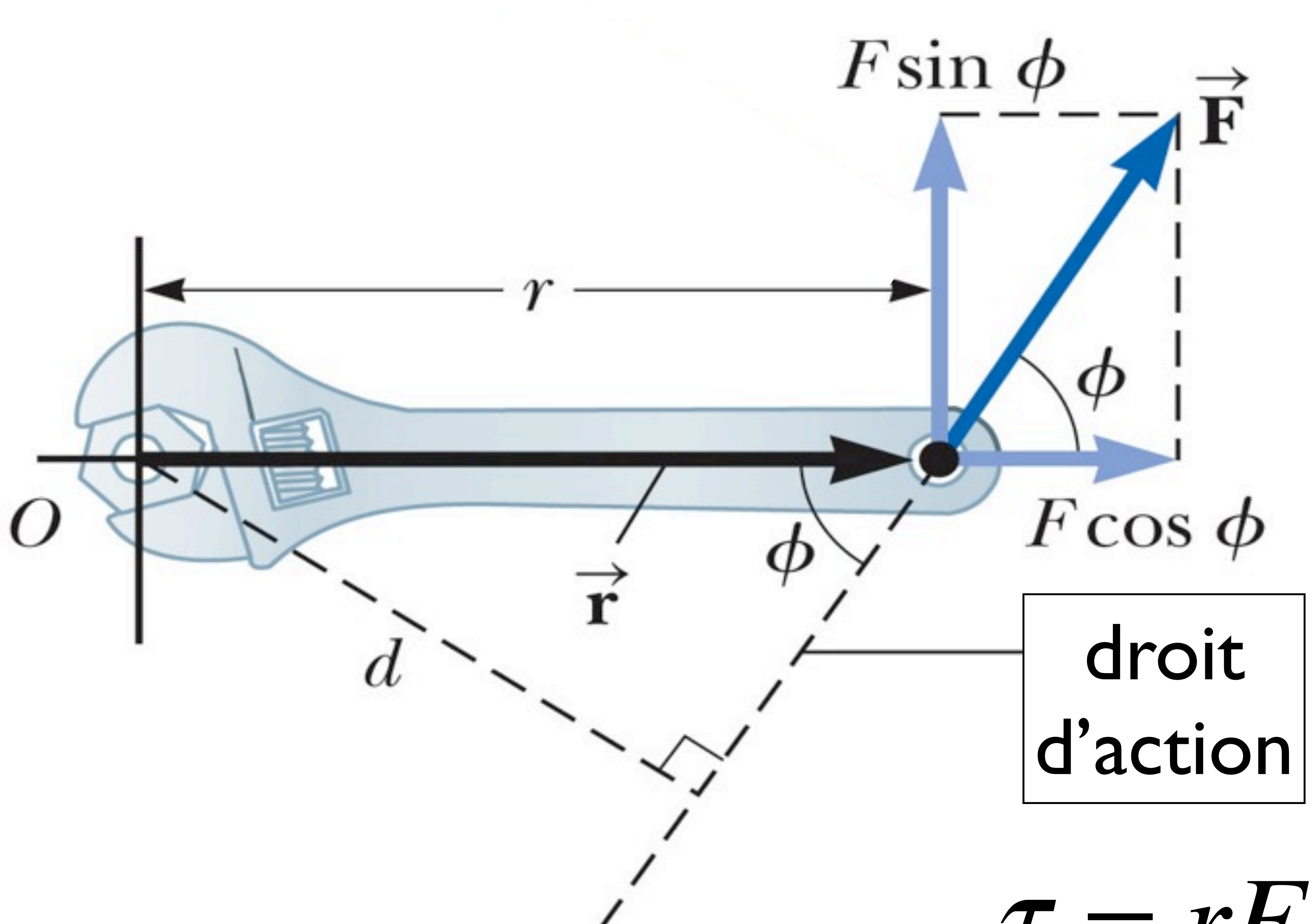
Moment de force, équilibre d’un corps rigide. Rotations

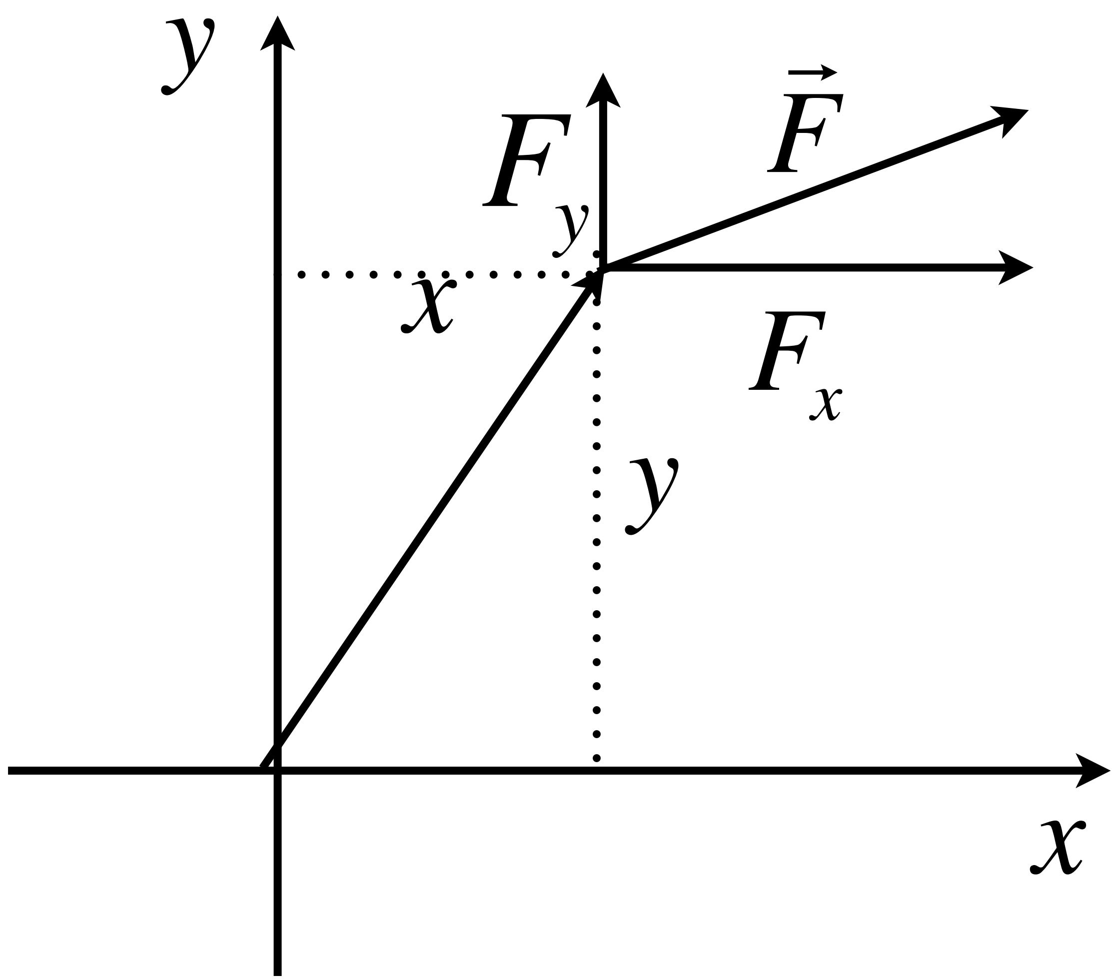
1. Rotation
    1. calcul du moment d’inertie
    2. conservation du moment angulaire : pirouette, étoile à neutrons, pointage de satellites
    3. Stockage d’énergie
2. Gravitation

Moment de force



$$\tau = rF\sin\phi$$

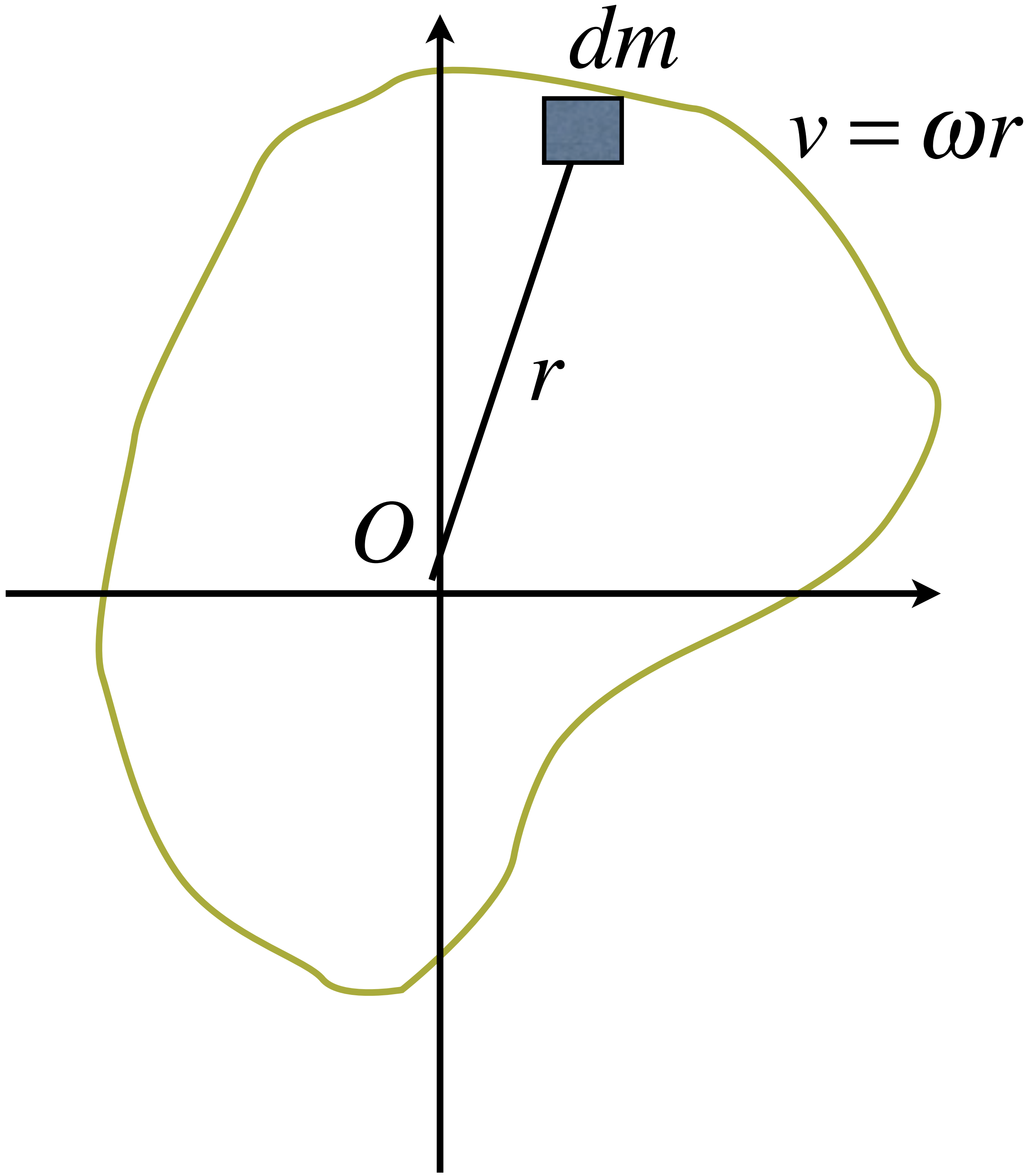
Le moment de force comme produit vectorielle



$$\tau = xF_y - yF_x$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Energie cinétique d'un objet rigide en rotation



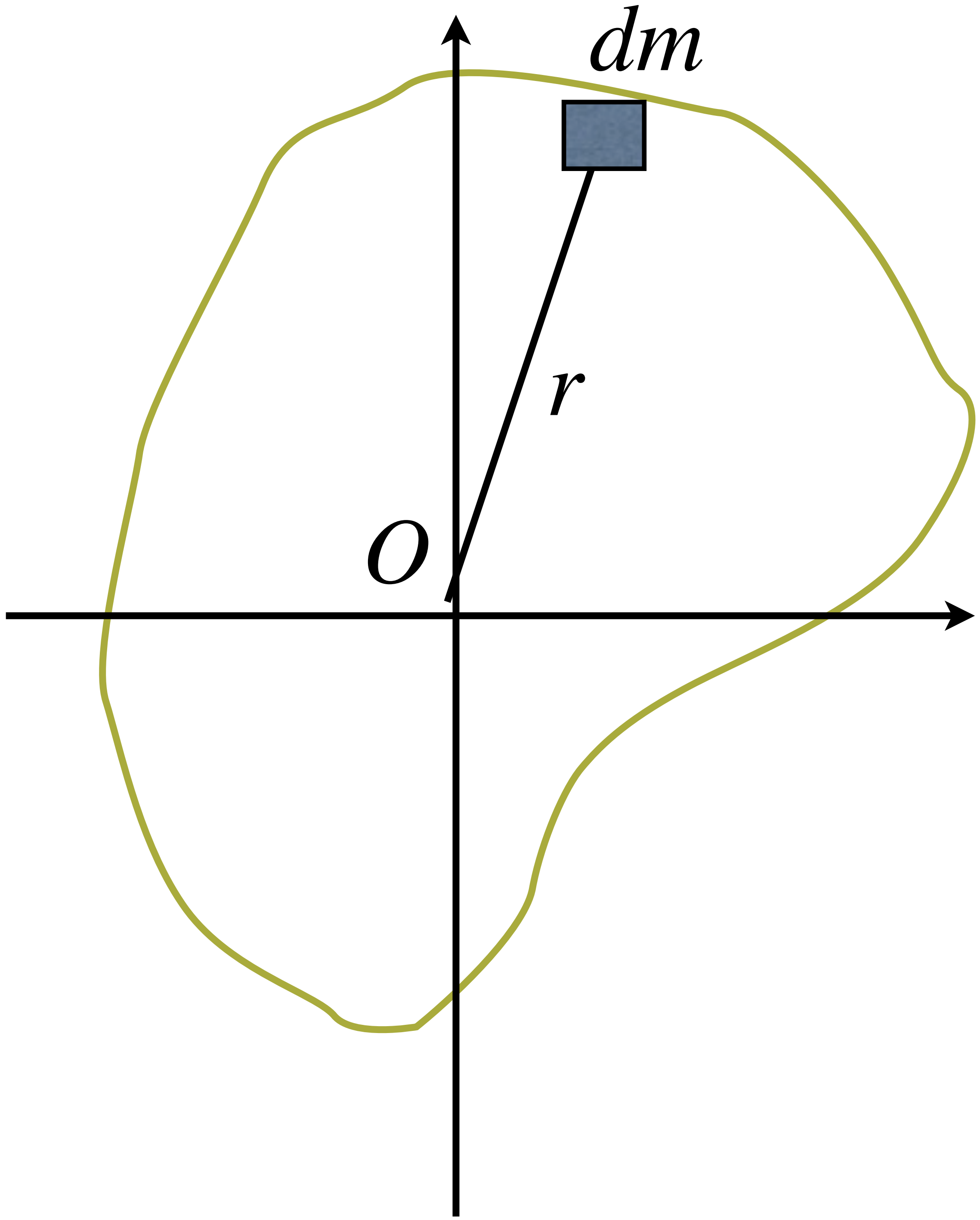
$$dE = \frac{1}{2} dm v^2 = \frac{1}{2} dm r^2 \omega^2$$

$$E = \int dE = \frac{1}{2}\left(\int r^2\, dm\right)\omega^2$$

$$E = \frac{1}{2} I \omega^2$$

$$\left(E = \frac{1}{2} m v^2 \right)$$

Moment d'inertie

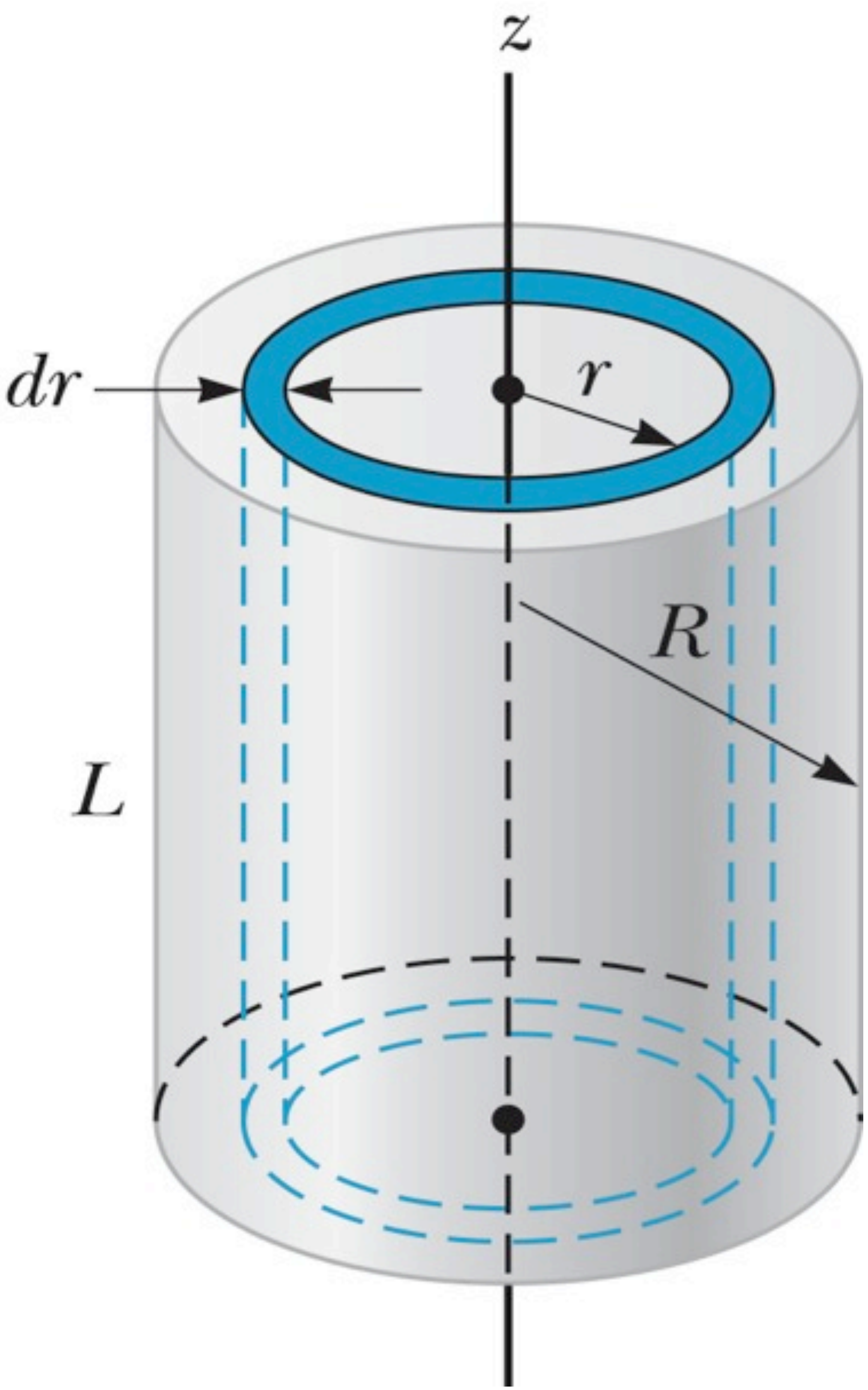


La quantité

$$I = \int r^2 \, dm$$

s'appelle moment d'inertie. Il joue la même rôle dans la déscription de la rotation que la masse dans la déscription du mouvement linéaire

Calcul du moment d'inertie II



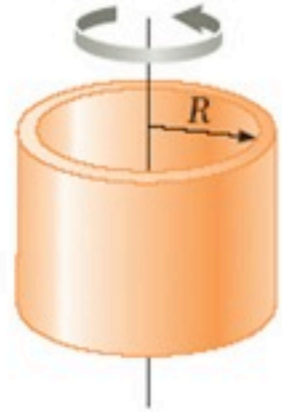
$$dm = 2\pi r dr L\rho$$

$$I_{cg} = \int r^2\, dm = 2\pi L\rho \int_0^R r^3\, dr$$

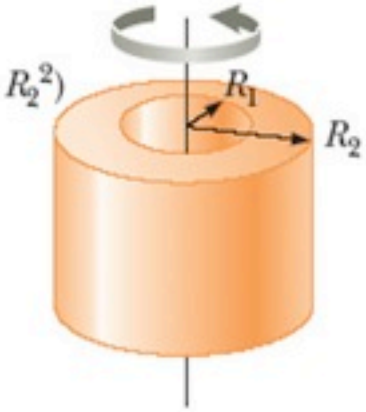
$$I_{cg} = \frac{1}{2} MR^2$$

TABLE 10.2 *Moments of Inertia of Homogeneous Rigid Objects with Different Geometries*

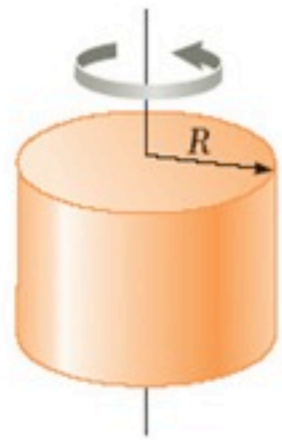
Hoop or thin cylindrical shell
$I_{CM} = MR^2$



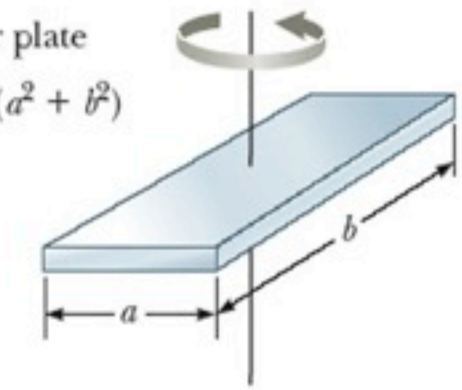
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1^{\,2} + R_2^{\,2})$



Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$

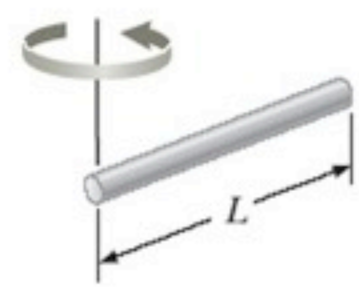


Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$

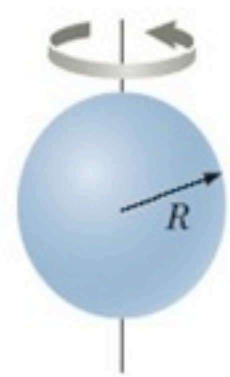


Long, thin rod with rotation axis through center
$I_{CM} = \frac{1}{12}ML^2$

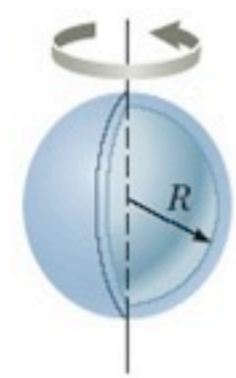
Long, thin rod with rotation axis through end
$I = \frac{1}{3}ML^2$



Solid sphere
$I_{CM} = \frac{2}{5}MR^2$

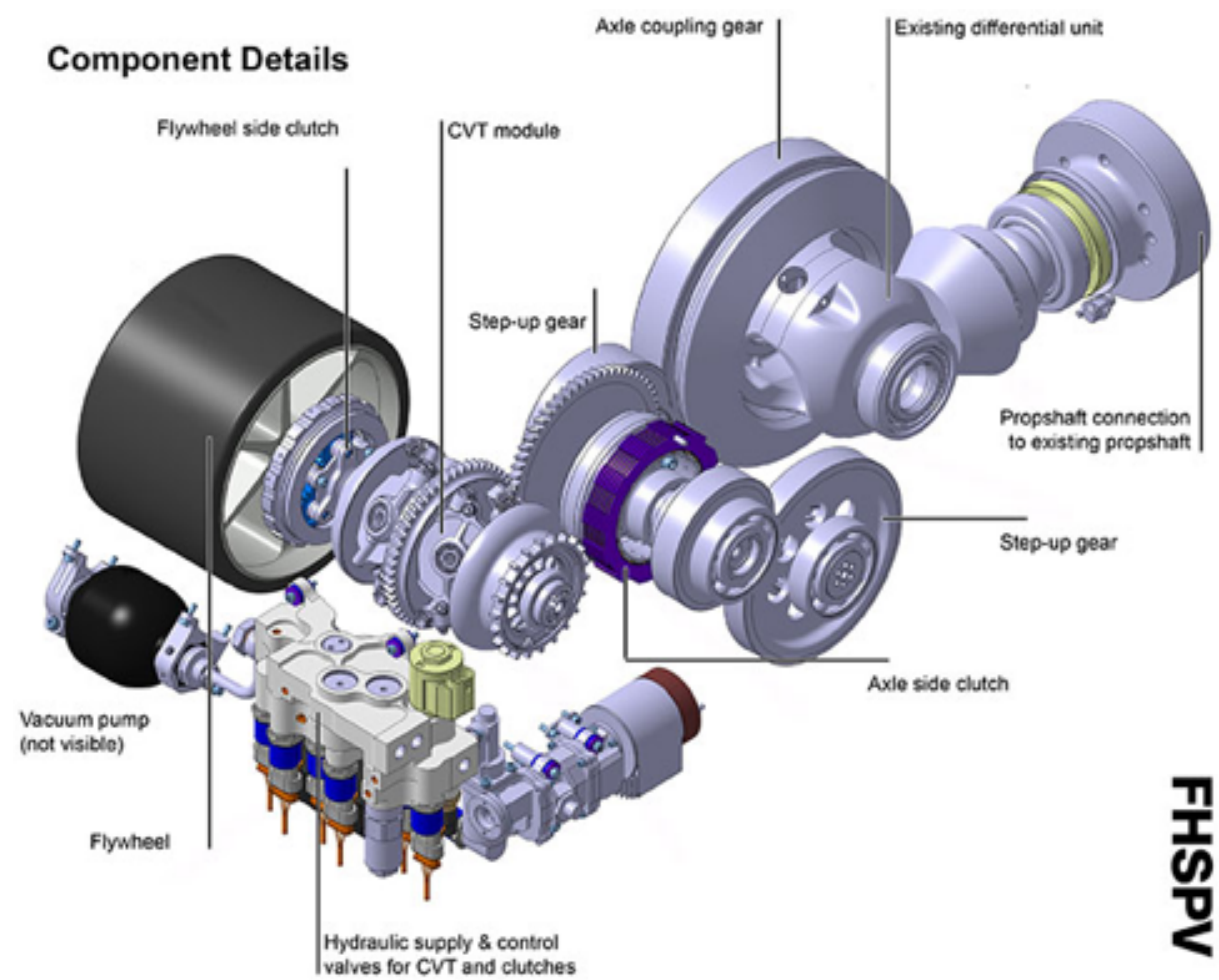


Thin spherical shell
$I_{CM} = \frac{2}{3}MR^2$



Stockage d'énergie en rotation

- KERS
- gyrobus

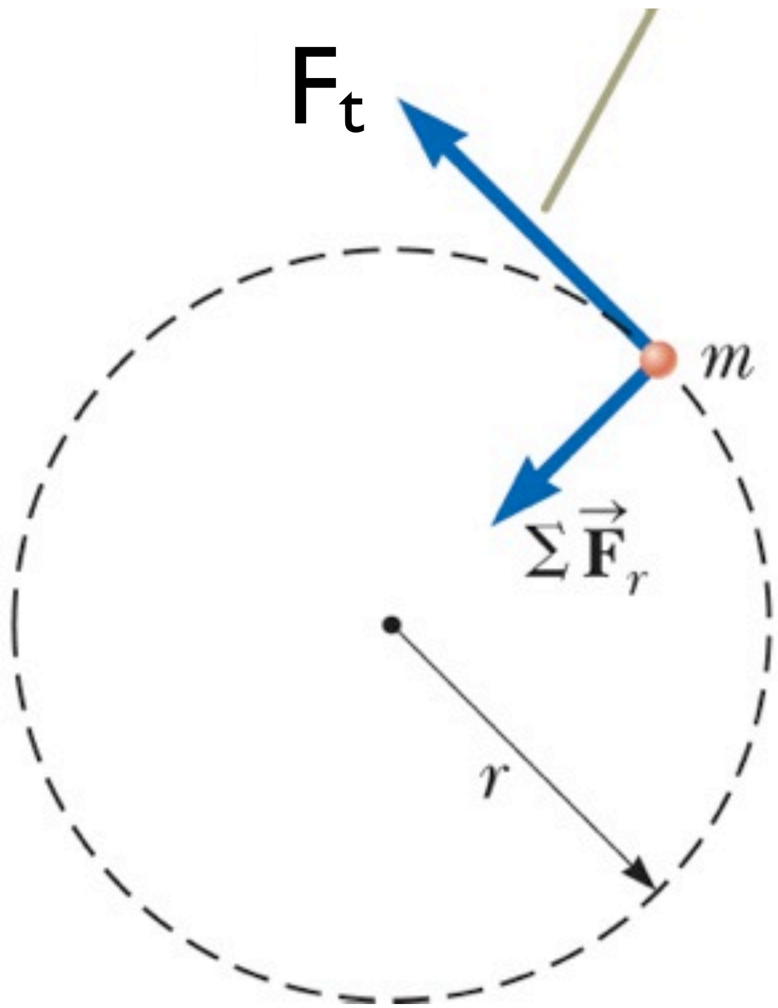


Gyrobus

M=1500 kg
d=1.6 m
max 3000 tours/min

E=...

Moment de force et accélération angulaire



$$F_t = m a_t$$

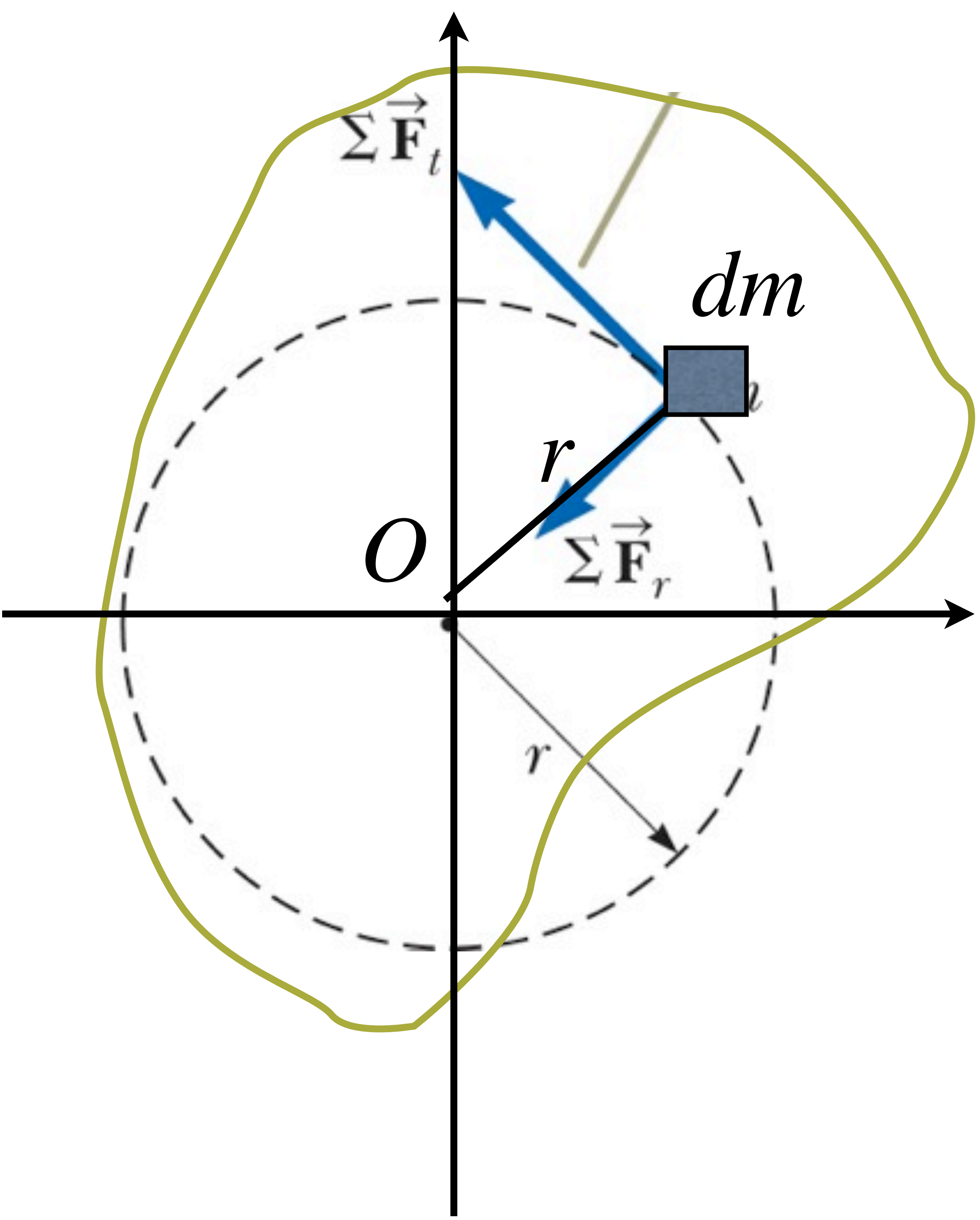
$$F_t r = m a_t r = m(\alpha r) r$$

$$F_t r = m r^2 \alpha$$

$$\tau = I\alpha$$

$$F = ma$$

Moment de force et accélération angulaire



$$dF_t = dma_t$$

$$dF_t r = dma_t r = dm(\alpha r)r$$

$$d\tau = dmr^2\alpha$$

$$\boxed{\tau = I\alpha}$$

Conservation du moment angulaire

$I\omega = const$ en absence de moments de force

http://www.youtube.com/watch?v=UnSYQIfiR6o

Etoile à neutrons

http://upload.wikimedia.org/wikipedia/commons/3/30/Crab_Lucky_video2.gif

$$I\omega = kMR^2 \frac{2\pi}{T} = const$$

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^2$$

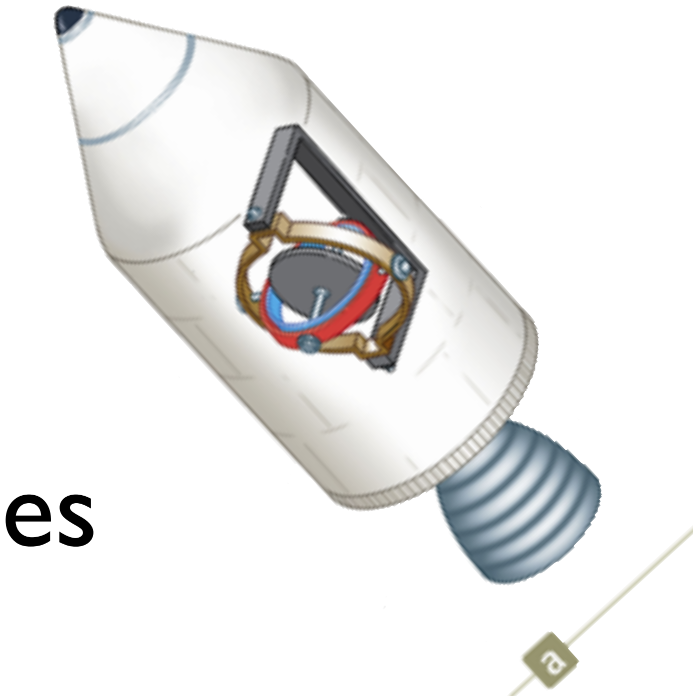
$R_1 = 1.6E4\text{km}$

$R_2 = 1.6\text{km}$

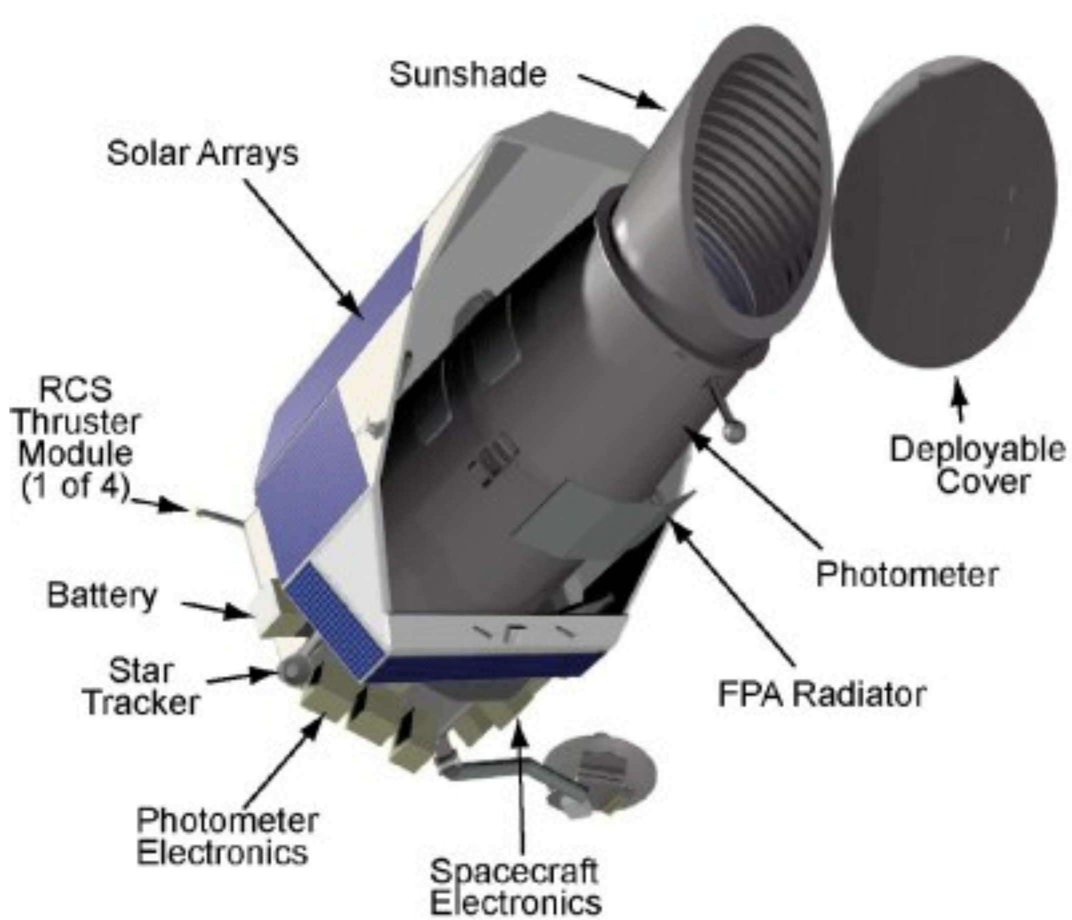
$T_1 = 26\text{d}$

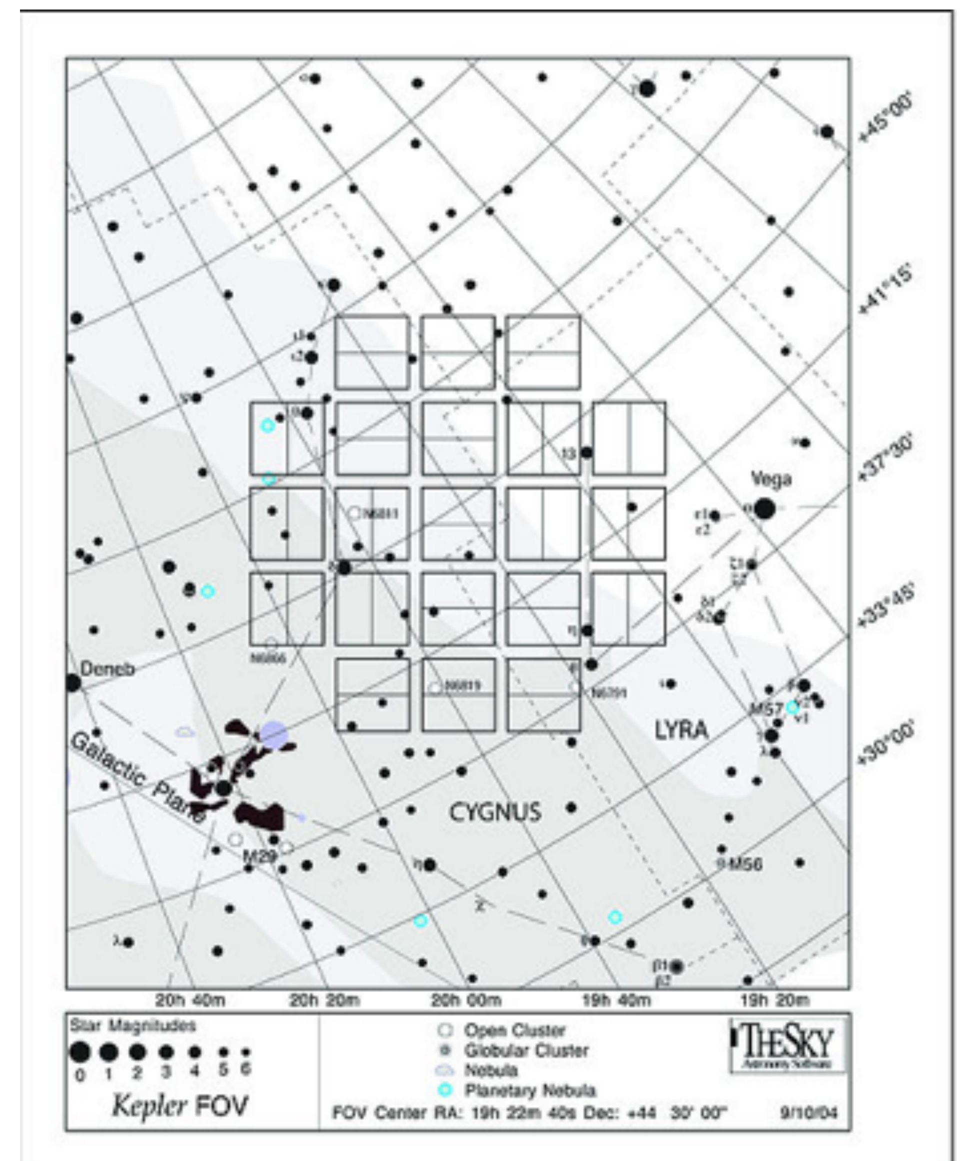
$T_2 = ?$

Contrôle d'attitude



- Roue de réaction pour orienter des satellites





http://kepler.nasa.gov/mission/